Statics

Coulombs law

The force F on a point charge q_1 at the point $\mathbf{r_1}$ caused by a point charge q_2 at the point $\mathbf{r_2}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 |r_1 - r_2|^2} \frac{(r_1 - r_2)}{|r_1 - r_2|}$$

Electric Field Strength

From a point charge q in $\boldsymbol{r'}$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{q}{4\pi\epsilon_0 R^2}\boldsymbol{e}_R$$

From charge distribution

$$\boldsymbol{E}(\boldsymbol{r}) = \int \frac{1}{4\pi\epsilon_0 R^2}\boldsymbol{e}_R dq(\boldsymbol{r'}),$$

$$dq(\boldsymbol{r'}) = \begin{cases} \rho_{tot}(\boldsymbol{r'})dv' = \rho(\boldsymbol{r'}) + \rho_p(\boldsymbol{r'}))dv' \\ \rho_{tot,s}(\boldsymbol{r'})dS' = \rho_s(\boldsymbol{r'}) + \rho_{p,s}(\boldsymbol{r'}))dS' \\ \rho_l(\boldsymbol{r'})dl' \end{cases}$$

From point dipole $\boldsymbol{p} = p\boldsymbol{e}_z$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos(\theta)\boldsymbol{e}_r + \sin(\theta)\boldsymbol{e}_\theta)$$

From line charge ρ_l

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\rho_l}{2\pi\epsilon_0 r_c}\boldsymbol{e}_{r_c}$$

From dipole line $\boldsymbol{p}_l = p_l\boldsymbol{e}_x$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{p_l}{2\pi\epsilon_0 r_c^2}(\cos(\varphi)\boldsymbol{e}_{r_c} + \sin(\varphi)\boldsymbol{e}_\varphi)$$

Electrical Potential

$$\boldsymbol{E} = -\boldsymbol{\nabla} V$$

From pointsource q in $\boldsymbol{r'}$

$$V(\boldsymbol{r}) = \frac{q}{4\pi\epsilon_0 R}$$

From charge distribution

$$V(\boldsymbol{r}) = \int \frac{1}{4\pi\epsilon_0 R} dq(\boldsymbol{r'})$$

From point dipole $\boldsymbol{p} = p\boldsymbol{e}_z$

$$V(\boldsymbol{r}) = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\epsilon_0 r^3} = \frac{p\cos(\theta)}{4\pi\epsilon_0 r^2}$$

From line charge ρ_l

$$V(\boldsymbol{r}) = \frac{\rho_l}{2\pi\epsilon_0}\ln\left(\frac{1}{r_c}\right)$$

From dipole line $\boldsymbol{p}_l = p_l\boldsymbol{e}_x$

$$V(\boldsymbol{r}) = \frac{p_l}{2\pi\epsilon_0}\frac{\cos(\varphi)}{r_c}$$

Electrical flow density

Where $\boldsymbol{D}$ is defined by $\boldsymbol{\nabla D} = \rho$

Gauss law, where $\boldsymbol{e}_n$ is the unit normal to the volume surface pointing outwards $\boldsymbol{P}, \boldsymbol{E}$ and $\boldsymbol{D}$:

$$\begin{cases} \boldsymbol{D} = \epsilon_0\boldsymbol{E} + \boldsymbol{P} & \text{(valid generally)} \\ \boldsymbol{D} = \epsilon_r\epsilon_0\boldsymbol{E} \end{cases}$$

Polarization Charge

$$\rho_p = -\boldsymbol{\nabla}\cdot\boldsymbol{P} \qquad \text{space charge density}$$

$$\rho_{p,s} = \boldsymbol{e}_{n1}\cdot(\boldsymbol{P}_1 - \boldsymbol{P}_2) \qquad \text{surface charge density}$$

where the unit normal $\boldsymbol{e}_{n1}$ is directed from 1 to 2.

Boundary Conditions

$$\begin{cases} E_t \text{ continuous} \\ \rho_s = \boldsymbol{e}_{n2}\cdot(\boldsymbol{D}_1 - \boldsymbol{D}_2) \end{cases}$$

where ρ_s is free surface charge density and $\boldsymbol{e}_{n2}$ is directed from volume 2 to volume 1.

Electrostatic Energy

$$W_e = \frac{1}{2}\sum_i Q_i V_i$$

$$W_e = \frac{1}{2}\int \rho V \, dv$$

$$W_e = \frac{1}{2}\int \boldsymbol{E}\cdot\boldsymbol{D}\, dv$$

Maxwell's voltage

$$|\boldsymbol{T}| = \frac{1}{2}\boldsymbol{E}\cdot\boldsymbol{D} \qquad \boldsymbol{E} \text{ is a bisector to } \boldsymbol{e}_n \text{ and } \boldsymbol{T}$$

Torque on Electrical Dipole

$$\boldsymbol{T}_e = \boldsymbol{p}\times\boldsymbol{E}$$