Quantum Mechanics

Schrödinger Equation

$$H\boldsymbol{\psi}(\boldsymbol{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + \mathcal{U}(\boldsymbol{r})\right]\boldsymbol{\psi}(\boldsymbol{r},t) = i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}(\boldsymbol{r},t)$$

Where H is a hamiltonian operator. If H is time independent separation of variables gives:

$$\boldsymbol{\psi}(\boldsymbol{r},t) = \boldsymbol{\Phi}(\boldsymbol{r}) \cdot e^{-\frac{i}{\hbar}Et}$$

$$\left[-\frac{\hbar^2}{2m}\Delta + \mathcal{U}(\boldsymbol{r})\right]\boldsymbol{\Phi}(\boldsymbol{r}) = E\boldsymbol{\Phi}(\boldsymbol{r})$$

The general time dependent solution is:

$$\boldsymbol{\psi}(\boldsymbol{r},t) = \sum_n a_n \cdot \boldsymbol{\Phi}(\boldsymbol{r})e^{-\frac{i}{\hbar}Et}$$

Where a_n are found through the boundary conditions ($t = 0$):

$$a_n = \int \boldsymbol{\Phi}_n * (\boldsymbol{r}) \cdot \boldsymbol{\psi}(\boldsymbol{r}, t = 0)\, d^3r$$